

Spontaneous modulation of superconducting phase in Kitaev ladder

Sho Nakosai,¹ Yukio Tanaka,² Tai Kai Ng,^{3,4} and Naoto Nagaosa^{5,6}

¹Condensed Matter Theory Laboratory, RIKEN, Wako, Saitama, Japan

²Department of Applied Physics, Nagoya University, Nagoya, Aichi, Japan

³Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China

⁴Hong Kong Academy for Gifted Education, Shatin, Hong Kong, China

⁵RIKEN Center for Emergent Matter Science (CEMS), Wako, Saitama, Japan

⁶Department of Applied Physics, University of Tokyo, Bunkyo, Tokyo, Japan

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We study theoretically the two Kitaev chains put in parallel, i.e., Kitaev ladder, coupled by Josephson junction. The π -junction between the Majorana bound states at the ends of the chains competes with the usual Josephson coupling along the chain, and this frustration leads to the modulation of the phase difference of the superconducting order parameter between the two chains. We show that this modulation gives the double degeneracy of the ground states, which can be manipulated by external electric and magnetic fields.

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Introduction.— The Kitaev chain [1], one-dimensional (single channel) p -wave superconducting wire, is well known as a prototype of topological superconductors [2–5]. The prominent feature is that it hosts Majorana modes as topologically protected states at the ends of the system. They have zero energy, which are energetically separated from other states by the superconducting gap, and then can remain localized at the ends. Since Majorana modes carry half the degrees of freedom of ordinary fermion, two Majorana modes sitting on the two ends constitute one fermionic mode. The occupied and unoccupied states of the mode have the same energy, and the ground state is doubly degenerate. As the consequence of peculiar properties of Majorana states, two Josephson-coupled topological superconductors produce a fermionic Andreev bound state with energy $\epsilon(\phi)$ which has 4π periodicity [6, 7] in sharp contrast to 2π periodicity in usual Josephson junctions. This is due to the conservation of the fermion parity, i.e., the energy levels of states with even and odd fermion parity cross at $\phi = 2\pi$ and they come back to the initial states only when the phase runs additional 2π if there is no parity-mixing process.

Majorana states exhibit non-Abelian statistics [8–10] and, therefore, have potential application in topological quantum computation [11]. The implementation have been done in some experimental setups: (i) spin-orbit-coupled quantum nanowire proximitized to s -wave superconductors [12–18] and the recent progress in Ref. [19]; (ii) magnetic adatoms on s -wave superconductors [20–23]; and (iii) induced superconductivity in two-dimensional topological insulators [24–28]. Let us consider the two Kitaev chains aligned in parallel where a Majorana state sit on each end of each chain. If they are totally decoupled, Majorana fermions remain zero energy states, therefore the ground state degeneracy is 4. However, in general, a Majorana state sitting on an end can couple with other Majorana states on the same end of the neighboring chains. This lifts the degeneracy and the energy of the Majorana states become nonzero by the amount of the tunneling element.

In this paper, we theoretically study two parallel Kitaev

chains, i.e., Kitaev ladder shown in Fig. 1, and discuss the effect of the inter-chain Josephson coupling. The competing Josephson couplings, i.e., that between Majorana bound states and that between bulk states, bring about the modulation in the phase of the superconducting order parameter and double degeneracy of the ground states in contrast to the naive expectation that all the degeneracies are lifted. One can also find some interesting works on aligned Kitaev chains in Refs. [29–35]. The change in the Majorana edge mode for the two-dimensional array of Kitaev chains due to the spontaneous phase modulation was also discussed in Ref. [36]. In contrast to these earlier works, we focus here the double degeneracy and its manipulation in the Kitaev ladder.

Majorana bound states.— Figure 1 shows the system we are considering, which is the two superconducting chains with a finite tunneling between them. Particularly, our interest is in the case when these chains are topologically nontrivial phases. The topological feature is manifested in the emergence of Majorana states at the boundaries of the system depicted by solid spheres in Fig.1. The Kitaev chain model provides the simplest description of this physics, which is a one-dimensional p -wave superconducting system [1]. The Hamiltonian for two

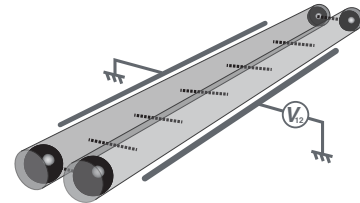


FIG. 1. Schematic picture of our model representing coupled two Kitaev chains which are connected through the single-electron tunneling shown in dotted lines. The solid spheres show Majorana states localized at the ends of the chains. V_{12} denotes the gate voltage relatively applied to chains 1 and 2.

Kitaev chains with tunneling connecting them reads

$$H = \sum_{\alpha=(1),(2)} \sum_{j=1}^N \left[\left(-tc_j^{\alpha\dagger} c_{j+1}^{\alpha} + \Delta_{\alpha} c_j^{\alpha} c_{j+1}^{\alpha} + \text{h.c.} \right) - \mu \left(c_j^{\alpha\dagger} c_j^{\alpha} - \frac{1}{2} \right) \right] + \sum_{j=1}^N \left(\lambda c_j^{(1)\dagger} c_j^{(2)} + \text{h.c.} \right), \quad (1)$$

where t is the nearest neighbor hopping strength, μ is the chemical potential, $\Delta_{\alpha} = |\Delta|e^{i\theta_{\alpha}}$ is the p -wave superconducting order parameter with the phase θ_{α} , and λ is the coupling between these two chains. We put parentheses for the indices indicating chains to avoid confusions. The electron operators c_j^{α} can be decomposed into Majorana operators γ_i^{α} in the following way

$$\begin{aligned} \gamma_{2j-1}^{\alpha} &= \exp\left(i\frac{\theta_{\alpha}}{2}\right) c_j^{\alpha} + \exp\left(-i\frac{\theta_{\alpha}}{2}\right) c_j^{\alpha\dagger}, \\ \gamma_{2j}^{\alpha} &= -i \exp\left(i\frac{\theta_{\alpha}}{2}\right) c_j^{\alpha} + i \exp\left(-i\frac{\theta_{\alpha}}{2}\right) c_j^{\alpha\dagger}. \end{aligned} \quad (2)$$

First, let us consider decoupled chains by setting $\lambda = 0$, and for simplicity, we choose particular set of parameters, $|\Delta| = t > 0$ and $\mu = 0$, where the system is in a topological phase. The Hamiltonian is written by using Majorana operators as

$$H_0 = it \sum_{\alpha} \sum_{j=1}^{N-1} \gamma_{2j}^{\alpha} \gamma_{2j+1}^{\alpha}. \quad (3)$$

In the ground state, $\gamma_{2j}^{\alpha} \gamma_{2j+1}^{\alpha}$ form dimmers depicted by the gray bonds in Fig. 2. γ_1^{α} and γ_{2N}^{α} are isolated Majorana modes which do not appear in the Hamiltonian, and thus these operators are related to zero energy states [1]. In general cases with different parameters, the corresponding states cannot be represented only by such completely isolated Majorana modes, but they still have zero energy and localized at the ends of the system until the gap in the bulk spectrum is closed at $|\mu| = 2|t|$. Their wavefunctions decay exponentially with the length scale of $\xi_M \sim t/|\Delta|$ (the lattice constant is set to be a unit).

Now, we turn on a finite single electron tunneling λ . The tunneling term in Eq. (1) can be written with respects to Majorana operators as

$$\begin{aligned} H_{\text{int}} &= \sum_i \lambda (c_i^{(1)\dagger} c_i^{(2)} + c_i^{(2)\dagger} c_i^{(1)}) \\ &= \frac{i\lambda}{2} \sum_i \left\{ \sin\left(\frac{\theta_1 - \theta_2}{2}\right) \left(\gamma_{2i-1}^{(1)} \gamma_{2i-1}^{(2)} + \gamma_{2i}^{(1)} \gamma_{2i}^{(2)} \right) \right. \\ &\quad \left. + \cos\left(\frac{\theta_1 - \theta_2}{2}\right) \left(\gamma_{2i-1}^{(1)} \gamma_{2i}^{(2)} - \gamma_{2i}^{(1)} \gamma_{2i-1}^{(2)} \right) \right\}. \end{aligned} \quad (4)$$

Here, we considered the interchain tunneling process only between the parallel sites. We will treat it as a perturbation. In the dimer limit (Eq. (3)), the effective Hamiltonian can be easily derived.

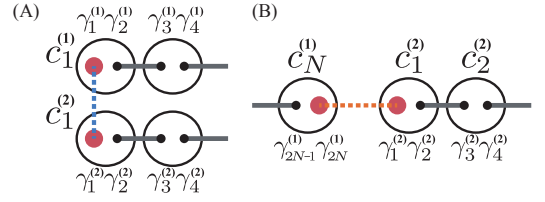


FIG. 2. (Color online) Two configurations of the coupling between Majorana edge states; parallel alignment (A) and head-to-tail configuration (B). The solid circles represent complex fermions which are decomposed into two Majorana states shown in black dots. The isolated Majorana states are colored with red.

Let us start with $\gamma_n^{(1)} \gamma_m^{(2)}$ with $n, m \neq 1, 2N$, i.e., the Majorana fermions participating in the dimer formation. For this coupling, there is a finite gap in the unperturbed Hamiltonian, and hence the second order perturbation theory can be applied. A straightforward calculation gives the perturbation energy $-(\lambda^2/16t) \cos(\theta_1 - \theta_2)$ except a constant energy shift. This dependence on the phase difference is expected for the ordinary Josephson coupling in superconducting junctions.

For $\gamma_n^{(1,2)}$ with $n = 1, 2N$, there is no unperturbed Hamiltonian, and hence the degenerate perturbation theory must be applied. Picking up terms which includes only these Majorana fermions in Eq. (4) are

$$\begin{aligned} &\frac{i\lambda}{2} \sin\left(\frac{\theta_1 - \theta_2}{2}\right) \left(\gamma_1^{(1)} \gamma_1^{(2)} + \gamma_{2N}^{(1)} \gamma_{2N}^{(2)} \right) \\ &= \lambda \sin\left(\frac{\theta_1 - \theta_2}{2}\right) (a^\dagger a + b^\dagger b - 1) \end{aligned} \quad (5)$$

where we have introduced (complex) fermion operators as $a = (\gamma_1^{(1)} + i\gamma_1^{(2)})/2$, $b = (\gamma_{2N}^{(1)} + i\gamma_{2N}^{(2)})/2$. Therefore, the ground state energy becomes $-\lambda |\sin(\frac{\theta_1 - \theta_2}{2})|$ by appropriate choice of $|0\rangle_i$ and $|1\rangle_i$ labeled by $i = a, b$ satisfying $a|0\rangle_a = 0$ and $|1\rangle_a = a^\dagger|0\rangle_a$ (and the same for b).

Of crucial importance is the dependence on the phase difference, which is the sinusoidal form. This should be contrasted to the anomalous Josephson coupling of Majorana states discussed in, for example, Refs. [6, 7]. The configuration there is depicted in Fig. 2 (B), and the coupling arises from the tunneling term $(\lambda c_N^{(1)\dagger} c_1^{(2)} + \text{h.c.})$ which results in $i\lambda \cos((\theta_1 - \theta_2)/2) \gamma_{2N}^{(1)} \gamma_1^{(2)}$. One can understand this form by looking at the last term in Eq. (4). The characteristic 4π periodicity is originating from the factor $1/2$ in the cos function.

GL theory of the superconducting phases.— We consider the GL theory of the phases of the superconducting order parameter based on the above result. In a system with an infinite length, the Hamiltonian is written as

$$\begin{aligned} H &= \int_0^L dx \left\{ \rho_s \left[(\partial_x \theta_1)^2 + (\partial_x \theta_2)^2 \right] - J \cos(\theta_1 - \theta_2) \right\} \\ &\quad - \lambda \left| \sin\left(\frac{\theta_1 - \theta_2}{2}\right) \right|_{x=0,L} \end{aligned} \quad (6)$$

where $J = \lambda^2/16t$ and ρ_s is the superfluidity density, i.e., the stiffness of the phase. We introduce the center of mass phase θ and the phase difference ϕ as $\theta_1 = \theta + \phi/2$ and $\theta_2 = \theta - \phi/2$. The solutions can be obtained by the ordinary stationary equation for both fields θ and ϕ . Since it is obvious that θ takes a constant value, we set it to be 0. As for the phase difference ϕ , we obtain the sine-Gordon equation

$$\partial_x^2 \phi - J \sin \phi = 0, \quad (7)$$

The boundary term in Eq. (6) will induce the deviation from the trivial solution $\phi = 0$, which can be treated by the part of the solution to Eq. (7) as given by

$$\phi(x) = \pm 4 \tan^{-1} \left[e^{(x-x_0)/\xi_\phi} \right] \quad (8)$$

with the assumption that $\phi(x \rightarrow -\infty) = 0$, $\phi(x \rightarrow \infty) = \pm 2\pi$, and $(d/dx)\phi(x \rightarrow \pm\infty) = 0$. This solution has a kink at $x = x_0$ which is fixed by the competition between the bulk term and boundary term in Eq. (6). The slope of the kink is determined by the typical length scale $\xi_\phi = \sqrt{\rho_s/J}$. The boundary condition at the ends of the system becomes

$$\left[\partial_x \phi \pm \lambda \cos \frac{\phi}{2} \right]_{x=0, L} = 0. \quad (9)$$

arising from the last term in Eq. (6). The point is that the phase difference has a finite value at the vicinity of the ends to minimize the energy due to the Majorana Josephson coupling whereas the phase difference in the bulk is close to 0 as in a usual Josephson junction. We emphasize that this spatial modulation of the phases along the chains originates from the zero energy Majorana bound states arising as topologically protected states. The position of the kink in the solutions Eq. (8), i.e. x_0 , is pinned by this boundary condition to a particular value with which

$$\phi(x=0) = \pm 4 \tan^{-1} \left(\sqrt{1 + \frac{16J}{\lambda^2}} - \frac{4\sqrt{J}}{\lambda} \right) \quad (10)$$

is satisfied. Eventually, the solutions to Eq. (7) with open boundaries are obtained as

$$\phi(x) = \pm 4 \left\{ \tan^{-1} \left[e^{-(x-x_0)/\xi_\phi} \right] \pm \tan^{-1} \left[e^{(x-x_0-L)/\xi_\phi} \right] \right\}, \quad (11)$$

where the ones with the plus sign in the curly bracket have the same sign at the two ends (see the left panel in Fig. 3) and the ones with the minus sign have the opposite signs (the right panel). We call them as kink-antikink and kink-kink solutions, respectively. Here we assume that the solutions are approximated simply by superpositions of two kink solutions. We evaluate the corresponding energies by substituting above solutions into Eq. (6). It turns out that the kink-antikink solutions result in lower energy. The gap in energy depends on the stiffness of the superconducting order parameter and decreases exponentially as $\exp(-L/\xi_\phi)$. The consequent supercurrent by the spatial modulation of the phase is calculated

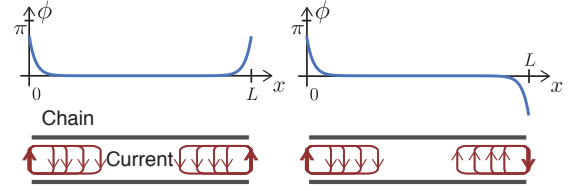


FIG. 3. (Color online) Schematic depiction of kink-antikink (left) and kink-kink (right) configurations of the phase difference and emerging supercurrent flowing in the system due to the modulation in the phase. The kink-antikink configuration has lower energy.

by using the expression $j = \delta H / \delta \phi$ with given configurations Eq. (11) as [37]

$$j_{\text{bulk}}(x) = \mp 2J \operatorname{sech}^{-1} [f_{J,\lambda}(x)] \tanh [f_{J,\lambda}(x)], \quad (12)$$

$$j_{\text{end}} = \pm \left(\frac{8J^{\frac{3}{2}}\lambda}{16J + \lambda^2} + \frac{2\sqrt{J}\lambda}{\sqrt{16J + \lambda^2}} \right), \quad (13)$$

where

$$f_{J,\lambda}(x) = \sqrt{J}x - \sinh^{-1} \left(\frac{4\sqrt{J}}{\lambda} \right). \quad (14)$$

The distribution of the current is schematically depicted in the second row in Fig. 3. One can see that it can be regarded as the loop current on each half of the system. The amount of the current through the bulk Josephson coupling is compensated by those at the ends through the Majorana bound states. It is naturally expected that these loop current generate magnetic field. We simply approximate the induced field as magnetic dipoles sitting on the ends. The interaction between the dipoles are antiferromagnetic, and clearly seen from Fig. 3, it further reduces the energy of a kink-antikink configuration. In summary, Majorana Josephson coupling produces non-zero phase difference near the ends of the system with plus or minus sign resulting in a kink-kink or kink-antikink configuration. The kink-antikink configuration is energetically preferable.

Low energy physics.— Finally, we construct an effective theory describing the low energy physics in our model based on the above discussion. We introduce the following set of Pauli matrices; σ_i^z corresponding to the occupancies of a and b fermions and τ_i^z the sign of the phase difference ϕ at the left and right ends. The Majorana Josephson coupling is represented by

$$H_m = m(\sigma_a^z \tau_a^z + \sigma_b^z \tau_b^z), \quad (15)$$

where we set $m = \lambda \sin(|\phi(0)|/2)$ (see Eq. (10)). The interaction between kink configuration at the both ends can be embedded in

$$H_r = -r\tau_a^z \tau_b^z, \quad (16)$$

where r is a positive value since the kink-antikink configuration is energetically favored as discussed (note that such a

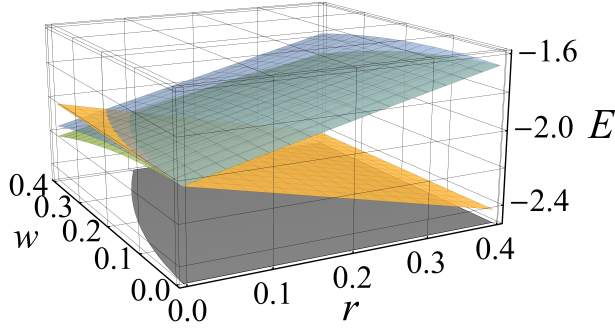


FIG. 4. (Color online) Eigenvalues of H_{eff} as functions of r and w . $m = 1$ and $|\Gamma| = 0.2$ are chosen. The yellow surface shows doubly degenerate eigenvalue. The shaded region on the bottom represents the area in the parameter space where this doubly degenerate ground state is realized.

configuration corresponds to the matrix element with the same eigenvalues of τ_i^z). r scales as $\exp(-L/\xi_\phi)$.

Another possible term comes from the overlap of the Majorana bound states at two ends of each chain. It is described by using complex-fermion operators a and b

$$\begin{aligned} H_w &= w(i\gamma_1^{(1)}\gamma_{2N}^{(1)} + i\gamma_1^{(2)}\gamma_{2N}^{(2)}) \\ &= w(a^\dagger b - b^\dagger a), \end{aligned} \quad (17)$$

where $w \propto \exp(-L/\xi_M)$. Equation (17) can be rewritten in terms of $\sigma_i^{x,y}$ as

$$H_w = w(\sigma_a^x \sigma_b^y - \sigma_a^y \sigma_b^x). \quad (18)$$

We also consider an additional non-trivial term expressing the macroscopic quantum tunneling with the matrix element Γ as

$$H_{\text{MQT}} = -\Gamma(\tau_1^+ + \tau_2^+) - \Gamma^*(\tau_1^- + \tau_2^-). \quad (19)$$

It describes quantum tunneling process connecting kink and antikink-configurations. The total effective Hamiltonian $H_{\text{eff}} = H_m + H_r + H_w + H_{\text{MQT}}$ is in the form of 16×16 matrix and the energy eigenvalues can be obtained by the diagonalization. Here m is considered to dominate other parameters since both r and w decay exponentially in the length of the system. Figure 4 shows four low-lying eigenvalues out of 16 eigenvalues of H_{eff} . Here we set $m = 1.0$ and $|\Gamma| = 0.2$. The shaded area shown in the bottom of the graph shows the set of parameters r and Γ with which the doubly degenerate eigenvalues (shown in the yellow curve) have the lowest values, i.e., the ground state is degenerate. Note that r and w have different length scales ξ_ϕ and ξ_M , then, we expect that in a system with $\xi_\phi > \xi_M$ the magnitude of r exceeds that of w and the ground state is degenerate. For example, in the experiment found in Ref. [22] they reported the localization length of the Majorana states are comparable to the size of an adatom on the substrate superconductor [38]. The states corresponding to the degenerate energy are ferromagnetic in

pseudospin of fermionic states $\langle \sigma_1^z \rangle = \langle \sigma_2^z \rangle$ and their configurations of the superconducting order parameter are superposition of two kink-antikink configurations. We can argue that by taking into account the phase degrees of freedom and by the virtue of anomalous Majorana Josephson coupling the ground state degeneracy holds even when the finite size effect and the quantum tunneling processes play roles.

Conclusion and outlook.— We investigate the role of modulation in the superconducting order parameter in two Kitaev chains put in parallel. When those are in the topological phase, single electron tunneling between Majorana states sitting at the ends is allowed and gives anomalous Josephson coupling, which results in spatially modulated superconducting phase. Due to this modulation, the ground state are doubly degenerate even though we take into account the overlap of Majorana wavefunctions. This double degeneracy can act as a qubit, and its manipulation is an important issue. As discussed above, the kink-antikink solutions don't have the net magnetic field, and hence cannot be biased by the uniform external magnetic field. However the local magnetic field applied only on one end of the system lifts the degeneracy.

The other means to control the state is to put the potential difference $V_{12}(t)$ between the two chains, which modulates the phase of the tunneling matrix element Γ in Eq. (19) between the two degenerate configurations of the superconducting order parameter. This comes from the canonical conjugate relation between the charge difference $Q_1 - Q_2$ between the two chains and the phase difference ϕ of the superconducting order parameter. $V_{12}(t)$ is coupled to $Q_1 - Q_2$. After integrating over $Q_1 - Q_2$, we obtain the term $iV_{12}d\phi/dt$ in the Lagrangian, which put the phase factor $\exp\left(icV_{12}\int_0^L dx \bar{\phi}(x)\right)$ to Γ (c : a constant depending on the capacity of the system, $\bar{\phi}(x)$: the difference of $\phi(x)$ between the two stable kink configurations). Combining these two external fields, one can manipulate the dynamics of the qubit.

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